THE INTERPLAY BETWEEN GEOSTATISTICAL PRIOR INFORMATION AND MODELING ERROR IN SEISMIC DATA

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• MOTIVATION
• FORWARD MODEL
• PRIOR MODEL
• FORWARD MODELING ERROR
• INVERSION RESULTS
• SUMMARY
Motivation

• Linearised AVO inversion is popular
• Linear forward (g)
  \[ d = g(m) \rightarrow d = Gm \]
  • Computationally efficient
  • Analytical solutions to inverse problem

• Assumptions
  • Prior model is Gaussian

• Is the subsurface really Gaussian?
  • What kind of errors do we introduce in the forward?
    • Can they be quantified?
    • Can we account for these errors?
Forward model
The basics

• Seismic signal
  • Approximated by convolutional model
    \[ S(t) = W(t) * R(t) \]

  \( W(t) \): Seismisk wavelet

  \( R(t) \): Seismisk reflektion koefficienter (seismisk forward)

• Reflection coefficients can be calculated by Zoeppritz equations

• Or approximated
  • E.g Aki & Richards, Shuey
  • Linear: Buland & Omre\(^1\)

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Buland & Omre - Linear forward

\[ d_{AVO} = W \cdot R + e = W \cdot G \cdot m' + e \]

- **W**: Wavelet matrix
- **R**: Seismic reflection coefficients
- **G**: Linear forward
- **m**: Model parameters

- Requirements: log-Gaussian prior distribution
  \[ m(t) = [\ln \alpha(t), \ln \beta(t), \ln \rho(t)]^T \]

- Benefits: Linear inversion
- Problems: Introduce forward modeling error?
Prior model
Prior model - Two cases

• ‘Best-case’
  • Continuous Gaussian vector field
    • Methodology of Buland & Omre (2003)
    • Correlation between model parameters (cor = 0.7)
    • Spatial correlation (range = 5 ms)
    • ???? + 2 other covariance types
      • Cov = '1 Sph(1,0,5e-3)'
      • Cov = '1 Exp(1,0,5e-3)'

• ‘Worst-case’
  • Discontinuous
    • Truncated pluri-Gaussian model
    • 4 layers
    • Covariance models
      • Cov = '1 Gau(1,0,5e-3)'
      • Cov = '1 Exp(10,0,5e-3)'

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Reference model
Forward modeling error
Forward modeling error

\[ S_{\text{error}}(t) = S_{\text{zoep}}(t) - S_{\text{app}}(t) \]

\[ S_{\text{error}}(t) = W \ast R_{\text{zoep}}(t) - W \ast R_{\text{app}}(t) \]

- Obtaining a sample of forward modeling error
  - A realisation of prior model
  - Calculate two forward responses: true and approx.
  - Convolve both seismic reflection series with wavelet

- A realisation of forward modeling error for a given prior model
One realisation
Forward modeling error sample

- 1000 realisations
  - Both prior models
- Standard deviation of modeling error sample at different incident angle

- Comparison
  - 1000 realisations of seismic AVO data calculated using Zoeppritz equations as forward
Quantifying modeling error
Quantifying modeling error

- Adapt methodology proposed by Hansen et. al. (2014)\(^2\)
- Estimate mean

\[
\mathbf{d}_{T_{\text{app}}} = [\mathbf{d}^1_{T_{\text{app}}}, \mathbf{d}^2_{T_{\text{app}}}, \ldots, \mathbf{d}^N_{T_{\text{app}}}] \\
\text{where } \quad d^i_{T_{\text{app}}} = \frac{1}{N} \sum_{i=1}^{N} (D_{\text{ex}}^{i,j} - D_{\text{app}}^{i,j})
\]

- and covariance modeling error

\[
\mathbf{C}_{T_{\text{app}}} = \frac{1}{N} \mathbf{D}_{\text{diff}} \mathbf{D}_{\text{diff}}' \quad \text{where } \quad \mathbf{D}_{\text{diff}} = [\mathbf{D}_{\text{ex}} - \mathbf{D}_{\text{app}} - \mathbf{D}_{T_{\text{app}}}]
\]

\[
\text{and } \quad \mathbf{D}_{T_{\text{app}}} = [\mathbf{d}'_{T_{\text{app}}}, \mathbf{d}'_{T_{\text{app}}}, \ldots, \mathbf{d}'_{T_{\text{app}}}].
\]

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Quantifying modeling error - Validation

- $N = 1000$ realisations
Quantifying modeling error - Covariance

- $C_{\text{Tapp1}}$: Estimated modeling error covariance
- $C_{\text{Tapp2}}$: Uncorrelated part ($\text{Diag}(C_{\text{Tapp1}})$)
- $C_{\text{Tapp3}}$: Constant variance ($\sigma^2 = 5e-5$)

- Sample each covariance matrix
  - Generate 1000 realisations

- Compare with actual modeling error
  - Histogram, frequency content, log likelihood distribution
Quantifying modeling error - Comparison

**Gaussian**

- Counts vs. Modeling error

**Discrete**

- Counts vs. Modeling error

**Frequency (Hz)**

- Magnitude vs. Frequency (Hz)

Legend:

- Actual modeling error
- $C_{tapp1}$ sample
- $C_{tapp2}$ sample
- $C_{tapp3}$ sample
Quantifying modeling error - likelihood

- Calculating likelihood that realisations originate from the estimated modeling error covariance ($C_{T_{app}}$) with the mean ($d_{T_{app}}$)

\[
\log(L(m)) = -0.5 \left( d_{test} - d_{T_{app}} \right)^t C^{-1}_{T_{app}} \left( d_{test} - d_{T_{app}} \right)
\]
Quantifying modeling error

Gaussian

Discrete

Actual modeling error $C_{\text{tapp}_1}$ sample $C_{\text{tapp}_2}$ sample $C_{\text{tapp}_3}$ sample

-10^8 -10^6 -10^4 -10^2 -10^0

-10^10 -10^5 -10^0

-800 -600 -400 -200 0

-800 -600 -400 -200 0
Linear AVO inversion
Linear AVO inversion - Setup

• Only possible for Gaussian prior
• Added white noise to reference data
  • SN ratio is constant for all incidence angles
  • $S$: Standard deviation of signal (zoeppritz)
  • $N$: Standard deviation of white noise
  • → increasing variance of noise for increasing offset
Linear AVO inversion - Posterior

• ‘Classic’ AVO inversion
  • Noise model: Uncorrelated white noise: $\Sigma_e$

• AVO Inversion accounting for modeling error
  • Noise model: Uncorrelated white noise: $\Sigma_e + \text{estimated forward modeling error: } C_Tapp$
Linear AVO inversion

Bayesian AVO inversion: SN = 5

Bayesian AVO inversion with modeling error taking into account: SN = 5
Posterior - Likelihood

Modeling error

Modeling error accounted for

SN = 0.5

SN = 1

SN = 2

SN = 4

SN = 15

SN = 20

\( \chi^2 \)
In summary

• Choice of prior model determines level of forward modeling error
• Significance of forward modeling error increases with incidence angle
• We are able to quantify the modeling error with a gaussian covariance model
• This enables accounting for modeling errors in linear AVO inversion for Gaussian priors